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Effective Area of Satellite-borne Antennas for Radio Astronomy

1. Introduction

The use of satellite antennas for precision measurements of polarized or unpolarized radiation in space requires a consistent quantitative antenna theory. For most practical antenna applications, only a rough qualitative knowledge is required. However, when the radio astronomer is concerned with interpreting the numerical values of antenna measurements, a thorough understanding of the operation of the antenna is required.

This paper is concerned with some elementary problems in the use of satellite antennas. These problems are covered under the general topic of "determining the effective area" of a receiving antenna. In more elementary terms, the radiation impinging on the antenna must be related to measurements at the antenna terminals. For example, the relation between the power density in a randomly polarized field near the antenna and the power dissipated in a load resistor connected to the antenna terminals must be established.

The "effective area problem" requires accurate values of the antenna parameters, e.g. driving point impedance. Accurate antenna parameters have been computed for only a very few antenna types. Further complications are introduced by the nature of the medium, such as a satellite antenna operating in a lossy plasma or in a plasma containing a magnetic field.

2. Effective Area of Linear Antenna in an Infinite Isotropic Medium

The effective area $A(m^{-2})$ of an antenna immersed in an infinite medium with an incident wave of power (i.e. flux) density $S(wm^{-2})$ is given by

$$A = P_{RL}/S \tag{2-1}$$

where P_{RL} = real power dissipated in a load impedance connected to the antenna terminals.

As an example, consider the system given in Fig. 2-1.

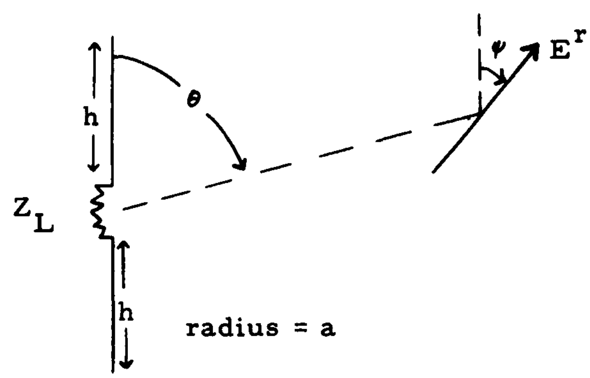

Fig. 2-1. Arbitrarily Oriented Plane Wave Impinging on a Receiving Antenna

A linear antenna of half-length h is placed in an arbitrarily oriented plane wave field. The angle that the propagation direction makes with the antenna is θ and the tilt angle between the incoming elective field $E^r(Vm^{-1})$ and the antenna is ψ. The load impedance Z_L consists also of the contributions due to the transmission line and to lumped impedances representing the effects of the junction at the antenna terminals [1]. The Thévenin equivalent circuit [2] of the receiving antenna aids in relating the incident electric field E^r to measurable quantities at the antenna terminals. The equivalent circuit is given in Fig. 2-2.

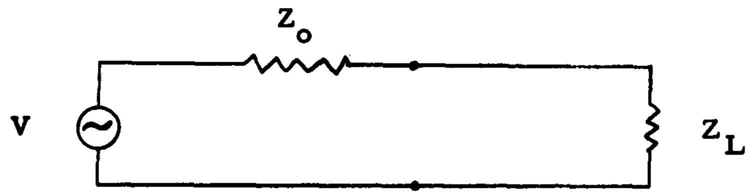

Fig. 2-2. Equivalent Circuit of Receiving Antenna

The equivalent generator voltage of the antenna V is given by [3] .

$$V = -(E^r \cos \psi)\ 2h_e(\theta) \tag{2-2}$$

In Eq. (2-2) the quantity $h_e(\theta)$ is the complex effective length of the receiving antenna. Numberical values of $h_e(\theta)$ are given by King [4] as a function of h, a, and θ. For an electrically small antenna,

$$h_e(\theta) = \frac{1}{2}\ h \sin \theta \tag{2-3}$$

for $(\beta_o h)^2 << 1$, where

$$\beta_o = \frac{2\pi}{\lambda_o}$$

and

$$\lambda_o = \text{free space wavelength .}$$

For an electrically small antenna, the equivalent voltage appearing at the antenna terminals becomes:

$$V = -hE^r \cos\psi \sin\theta \qquad (2\text{-}4)$$
$$\text{for } (\beta_o h)^2 << 1 \quad .$$

The above relation shows that for $(\beta_o h)^2 << 1$, the geometrical projection of the electric field multiplied by the half-length of the antenna yields the equivalent emf. In general, however, the complex effective length $h_e(\theta)$ must be fully taken into account.

Returning to the equivalent circuit in Fig. 2-2, we see that the Thévenin equivalent impedance is Z_o, the driving point impedance of the antenna. (N. B. for non-perfect conductors, the self-impedance must be included in Z_o.) This impedance is <u>complex</u> and a function both of the electrical length $\beta_o h$ and the radius $\beta_o a$ of the linear antenna. Tables of Z_o are given in the literature [5]. For an electrically short antenna ($\beta_o h < 1$), Z_o is approximately given by [6] :

$$Z_o(\beta_o) = \frac{\zeta_o \psi_{dl}}{6\pi(\Omega-3)} \; \frac{\beta_o^2 h^2}{\left[1 + \frac{\beta_o^2 h^2 F}{3}\right]} - j\left\{\frac{\zeta_o \psi_{dl}}{2\pi\beta_o h\left[1 + \frac{\beta_o^2 h^2 F}{3}\right]}\right\} \qquad (2\text{-}5)$$

where

ζ_o = characteristic impedance of free space

= 120π ohms ($\cong$ 377 ohms)

Ω = $2\,\ell n\,(2h/a)$

a = radius of linear antenna (see Fig. 2-1)

ψ_{dl} = $2\,\ell n\,(h/a) - 2$

F $\cong$ $1 + [1.08/(\Omega-3)]$.

The power dissipated in the load impedance Z_L of Fig. 2-2 is:

$$P_L = \frac{V^2}{(Z_L + Z_O)^2} = \frac{[2E^r \cos\psi\, h_e(\theta)]^2}{(Z_L + Z_o)^2} \cdot Z_L \quad . \tag{2-6}$$

Since $h_e(\theta)$, Z_L, and Z_o are complex, the power P_L is complex, i.e.

$$P_L = \mathrm{Re}P_L + j\,\mathrm{Im}\,P_L = P_{LR} + j\,P_{Li}$$

Re = real part

Im = imaginary part . (2-7)

For a short antenna ($\beta_o h < 1$), Z_o is primarily capacitive. For example, with $\beta_o h = 0.3$ and $\Omega = 10$, then

$$Z_o = 1.66 - j\,1274 \text{ ohms} \quad . \tag{2-8}$$

The maximum power transfer from the radiation field to the load impedance occurs when

$$Z_L = Z_o{}^* \tag{2-9}$$

where * denotes complex conjugate.

The condition in Eq. (2-9) corresponds to

$$R_L = R_o$$

and

$$X_L = -X_o$$

where

$$Z_o = R_o + jX_o$$

$$Z_L = R_L + jX_L \quad . \tag{2-10}$$

With Eqs. (2-9) and (2-10) in Eq. (2-6), the optimum received power $\overline{P}_{RL}$ is given by

$$\overline{P}_{RL} = \frac{[2E^r \cos\psi\, h_e(\theta)]^2}{4R_o} \tag{2-11}$$

The effective area for a plane wave source is found by relating the incoming power density S to P_{RL}. The power density in a plane wave is given by

$$S = \frac{1}{2}\,(\overrightarrow{E^r} \times \overrightarrow{H^*})\ \mathrm{wm}^{-2}\,(\mathrm{cps})^{-1} \tag{2-12}$$

where H is the magnetic field strength in webers per square meter.

The factor $\frac{1}{2}$ occurs in Eq. (2-12) as E^r and H are maximum amplitudes. The magnetic field vector $\overrightarrow{H}$ is orthogonal to $\overrightarrow{E^r}$ and both are related by the free space impedance, from which

$$/H/ = \frac{/E/}{\zeta_o} \tag{2-13}$$

or, with Eq. (2-13) in Eq. (2-12),

$$S = \frac{1}{2}\ \zeta_o\ \ /E^r/^2 \quad . \tag{2-14}$$

With Eqs. (2-14) and (2-6) in Eq. (2-1), the effective area of a linear antenna in an arbitrarily oriented linear polarized plane wave field is

$$A = \mathrm{Re}\left\{ 8 \left| \frac{\cos \psi h_e(\theta)}{Z_L + Z_1} \right|^2 \zeta_o Z_L \right\} .$$

3. The Linear Antenna in Circularly and Randomly Polarized Fields

The circularly polarized field is a special case of elliptical polarization. An elliptically polarized field may be represented by two spatially orthogonal linear fields in time quadrature. The voltage at the terminals of the antenna is a superposition of the individual fields. The determination of the parameters of the polarization ellipse is discussed in the literature 7 . In general, the polarization ellipse is time dependent [i.e. $\psi = \psi(t)$] . The time dependency will appear as a modulation of the received power at the rotation frequency of the ellipse. As a specific example, consider the circularly polarized field of Fig. 3-1.

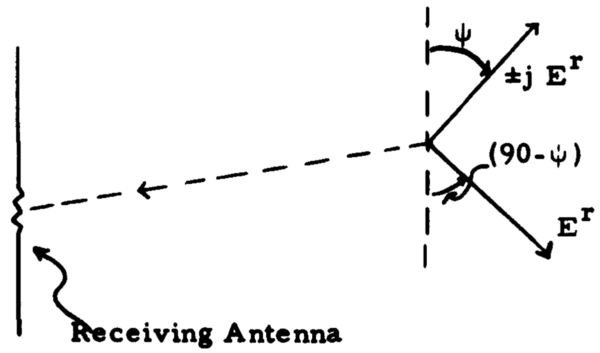

Fig. 3-1. Circularly Polarized Field Impinging on a Receiving Antenna

For the wave of Fig. 3-1, the equivalent voltage at the antenna terminal is

$$V = -[-E^r \sin\psi \pm jE_r \cos\psi] 2h_e(\theta)$$

$$= \mp j\, [E^r e^{\pm j\psi}]\; 2h_e(\theta) \qquad , \qquad (3\text{-}1)$$

where the upper sign is for right-hand polarization

The total power density S for the circularly polarized wave is

$$S = \frac{1}{2}\overrightarrow{E^r} \times \overrightarrow{H^*} + \frac{1}{2}(\pm j\overrightarrow{E^r}) \times \overrightarrow{H^*} \qquad (3\text{-}2)$$

or

$$/S/ = \frac{\sqrt{2}}{2\zeta_o} \quad /E^r/^2 \qquad . \qquad (3\text{-}3)$$

The real power P^c_{RL} at the terminals of the receiving antenna with a circularly polarized field is given by Eqs. (3-1), (3-3), (2-6), and (2. 7); then

$$P^c_{RL} = \sqrt{2}\; P_{RL} \qquad (3\text{-}4)$$

where P_{RL} is given by Eq. (2-6) (real part).

The power received at the antenna terminal with a randomly oriented field is obtained by averaging P_{RL} over all angles ψ, which gives

$$P^r_{RL} = \left\langle P_{RL} \right\rangle \qquad , \qquad (3\text{-}5)$$

where $\langle\ \rangle$ denotes the mean value. As $\langle \cos^2 \psi \rangle = \frac{1}{2}$, and with Eqs. (2-6) and (2-7) in Eq. (2-5),

$$P^r_{RL} = \frac{1}{2} P_{RL} (\psi = 0) \quad . \tag{3-6}$$

The variation of P^r_{RL} with respect to the propagation direction is taken into account in the definition of the effective area A. Equation (3-6) indicates that the power received from a randomly polarized wave is one-half of the power from a linearly polarized wave.

4. The Linear Antenna as a Probe for the Measurement of Temperature in a Black Body Enclosure

A thermodynamic study of an antenna immersed in a black body enclosure of temperature T[8] shows that the power developed in the load impedance of Fig. 2-1 is a direct measure of T. That is, the temperature of Z_L must equal T to insure thermodynamic equilibrium. The power density derived from the Rayleigh-Jeans approximation must be equal to the power density in the radiation field, or

$$S = \frac{1}{4} \zeta_o /E^r/^2 = 4\pi kT/\lambda^2 \ \mathrm{wm}^{-2}(c/s)^{-1} \quad , \tag{4-1}$$

where

ζ_o = 120 = 377 ohms

k = Boltzmann's constant

T = black body temperature in °Kelvin .

From Eq. (2-6), the square of the incident field is given by

$$/E^r/^2 = \frac{P^r_{RL}}{2h_e^2(\theta)} \cdot \frac{(Z_L + Z_o)^2}{Z_L} \tag{4-2}$$

where

$$P^r_{RL} = \text{power in load resistor due to a randomly polarized wave}$$
$$= \frac{1}{2} P_{RL}(\psi = 0) \quad .$$

With Eq. (4-2) in Eq. (4-1), the equivalent black body temperature is given by

$$T = \frac{\lambda^2}{32\pi k} \frac{1}{\zeta_o} P^r_{RL} \operatorname{Re}\left\{\frac{(Z_L + Z_o)^2}{Z_L h_e^2(\theta)}\right\} \tag{4-3}$$

Equation (4-3), for a matched load ($Z_L = Z_o$), reduces to

$$T = \frac{\lambda^2}{32\pi k} P^r_{RL} \operatorname{Re}\left\{\frac{4R_o^2}{(R_o - jX_o)h_e^2(\theta)}\right\} \quad . \tag{4-4}$$

5. Discussion

Previous theoretical work [9] has not been concerned with either the actual measurement problem or accurate values of the antenna parameters. For example, conventional antenna theory does not take account of the thickness of the antenna or the actual current distribution on the antenna. Since the driving point resistance of an antenna is small (i.e. for a short antenna), large errors are possible when "handbook" formulas are employed for radiation resistance. Furthermore, no previous discussion has considered properly the load impedance of the antenna or the matching of the antenna for a maximum transfer of power. It is hoped that the simple formulation of this paper will indicate that some care must be employed in interpreting measurements made on space probes. Further papers in this series will discuss in more detail the behavior of antennas in magnetoactive media.

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